THEORY OF THE ELECTRICAL DIFFUSION MEASUREMENT OF THE SPECTRAL CHARACTERISTICS OF A TURBULENT FLOW

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The diffusion to the surface is considered for a flat plate and a cylindrical wire in a pulsating flow of viscous incompressible liquid; the square of the modulus has been determined for the frequency response of an electrical diffusion transducer, which relates the spectral density of the velocity fluctuations (or else the friction fluctuations at the wall) to the spectral density of the mass fluctuations at the transducer.

One can examine the motion of a liquid via the heat transport rate, as in thermal anemometry, and also via the material transport rate.

A simple method of measuring the mass transport rate by reference to a monitor is to measure the limiting diffusion time of an electrochemical reaction; see [1, 2] for the details of the cell and the principle of the method of measuring mean flow characteristics. One can measure the frictional force of a flowing liquid at the wall of the channel by this method also [3, 4]. One determines the rate of redox reaction at the electrode, which fits flat with the surface, and to which an appropriate voltage is applied (Fig. 1). To reduce the ion concentration rate, and to eliminate migration current transport, one adds to the solution an excess of a second electrolyte whose ions do not participate in the reaction. If the voltage is high enough, the velocity determines the rate of diffusion transport of the monitored ions to the electrode surface. The diffusion current is related by the diffusion equation to the velocity and velocity fluctuations in the flowing electrolyte. The diffusion coefficient for the monitor ions is small, so a short electrode means that the thickness of the diffusion layer will be much less than the linear part in the velocity distribution for a laminar flow, or else it will be less than the viscous sublayer in a turbulent boundary layer. This feature considerably facilitates solution of the problems and allows one to use electrical diffusion to examine in detail the fluctuating flows directly near the wall.

Here we derive the frequency characteristics of a planar transducer used to measure the tangential stress at the wall, and also the same for the pulsating flow around a cylindrical transducer for the velocity. The frequency response of the linear system (a process at the monitor electrode is described by a linear equation) is determined as the response of the system to a harmonic system $A(\omega)\exp(i\omega t)$, i.e.,

$$H(\omega) = B(\omega)/A(\omega)$$

Here $H(\omega)$ is the frequency response and $B(\omega)$ is the amplitude of the signal at the output. In this case, the input signal is the pulsation amplitude of the velocity or friction at the wall, while the output signal is the mass flow rate pulsation.

1. Frequency Response of a Planar Transducer for Tangential Stress Pulsation Measurement

To determine the frequency response one needs to solve the transient-state diffusion equation for a boundary diffusion layer at the electrode:

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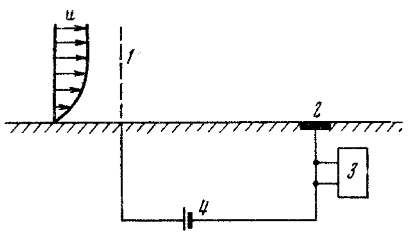

Fig. 1

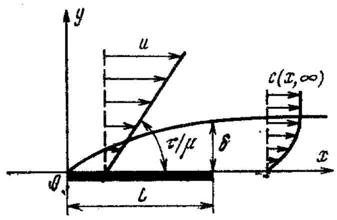

Fig. 2

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial y^2} \qquad (1.1)$$

subject to the boundary conditions

$$c(x, 0, t) = 0, \ c(0, y, t) = c(x, \infty, t) = c_\infty \qquad (1.2)$$

Here x and y are longitudinal and transverse coordinates, t is time, c is concentration of the reacting ions, u is electrolyte velocity, D is diffusion coefficient, and c_∞ = const. The speed in the diffusion layer is represented in the form of a sum:

$$u = u_0 + u_- \qquad (1.3)$$

where u_0 is the steady component of the velocity and u_- is the pulsating component of the velocity. We use the fact that the diffusion boundary layer is thin (Fig. 2), and employ the expression of [5] for the steady component of the velocity:

$$u_0 = \tau\mu^{-1}y \qquad (1.4)$$

Here τ is the tangential stress at the wall and μ is the dynamic viscosity; the pulsating component of the velocity is put as a simple harmonic:

$$u_- = \varepsilon u_0 \exp(i\omega t) \qquad (1.5)$$

where ω is the pulsation frequency and $\varepsilon = \text{const} \ll 1$. We introduce the dimensionless variables $x^*, y^*, t^*, u^*, c^*, \omega^*$ via

$$x^* = x/L, \quad y^* = y(\tau/\mu LD)^{1/3}, \quad t^* = t(\tau^2 D/\mu^2 L^2)^{1/3}$$
$$u^* = u(\mu^2/\tau^2 LD)^{1/3}, \quad c^* = c/c_\infty, \quad \omega^* = \omega(L^2\mu^2/\tau^2 D)$$

where L is the length of the monitor electrode; (1.1) and (1.2) take the following forms in these dimensionless variables:

$$\frac{\partial c^*}{\partial t^*} + u^*\frac{\partial c^*}{\partial x^*} = \frac{\partial^2 c^*}{\partial y^{*2}} \qquad (1.6)$$

$$c^*(x^*, 0, t^*) = 0, \quad c^*(0, y^*, t^*) = c^*(x^*, \infty, t^*) = 1 \qquad (1.7)$$

We seek the solution to (1.6)-(1.7) in the form of the sum

$$c^* = c_0^* + \exp(i\omega^* t^*)\, c_-^* \qquad (1.8)$$

where c_0^* is the steady-state component of the concentration, and c_-^* is the complex amplitude of the concentration fluctuation. If the velocity pulsations are small, the concentration pulsations will be small also, and then one can linearize (1.6) to determine the concentration distribution via the following two approaches. For the component c_0^* we have the problem

$$y^*\frac{\partial c_0^*}{\partial x^*} = \frac{\partial^2 c_0^*}{\partial y^{*2}} \qquad (1.9)$$
$$c_0^*(x^*, 0) = 0, \ c_0^*(0, y^*) = c_0^*(x^*, \infty) = 1$$

which may be solved after introducing the new independent variable $\eta = y^* x^{*-1/3}$, which gives

$$c_0^* = \int_0^{\eta} \exp(-\eta^3/9)\,d\eta \Big/ \int_0^{\infty} \exp(-\eta^3/9)\,d\eta \tag{1.10}$$

The steady-state mass flow rate per unit surface area of the transducer is

$$q_0 = \frac{D}{L}\int_0^L \left(\frac{\partial c_0}{\partial y}\right)_{y=0} dx = 0.806\,Dc_\infty(\tau/\mu LD)^{1/3} \tag{1.11}$$

The concentration pulsation amplitude is found from

$$i\omega^* c_-^* + y^* \frac{\partial c_-^*}{\partial x^*} + \varepsilon y^* \frac{\partial c_0^*}{\partial x^*} = \frac{\partial^2 c_-^*}{\partial y^{*2}} \tag{1.12}$$

subject to the uniform boundary conditions

$$c_-^*(x^*, 0) = c_-^*(x^*, \infty) = c_-^*(0, y^*) = 0 \tag{1.13}$$

The problem of (1.12) and (1.13) may be solved in sequence for small, large, and moderate pulsation frequencies.

Case of Low Pulsation Frequencies ($\omega^* \ll 1$). We represent c_-^* in the form of a series with respect to frequency:

$$c_-^* = \sum_{n=0}^{\infty} (i\omega^*)^n c_{n-}^* \tag{1.14}$$

We substitute (1.14) into (1.13) and equate equal powers of the frequency to get for c_{n-}^* a system of recurrence equations

$$\varepsilon y^* \frac{\partial c_0^*}{\partial x^*} + y^* \frac{\partial c_{0-}^*}{\partial x^*} = \frac{\partial^2 c_{0-}^*}{\partial y^{*2}} \tag{1.15}$$

$$c_{n-}^* + y^* \frac{\partial c_{n+1-}^*}{\partial x^*} = \frac{\partial^2 c_{n+1-}^*}{\partial y^{*2}} \tag{1.16}$$

where n = 0, 1, 2, ...

The solution to the first equation is the simple one

$$c_{0-}^* = \frac{1}{3}\varepsilon y^* \frac{\partial c_0^*}{\partial y^*} \tag{1.17}$$

This solution shows that the quasistationary part of the complex amplitude c_{0-}^* undergoes no phase shift relative to the superimposed signal, while the amplitude of c_{0-}^* is less by $\varepsilon/3$ than the mean mass flow rate. In (1.16) we make the substitution

$$c_{n-}^* = \varepsilon x^{*2n/3} c_{n+}^*(\eta) \tag{1.18}$$

which gives us a system of ordinary equations

$$\begin{gathered}\frac{d^2 c_{1+}^*}{d\eta^2} + {}^1\!/_3\,\eta^2 \frac{dc_{1+}^*}{d\eta} - {}^2\!/_3\, c_{1+}^* + \eta \frac{dc_0^*}{d\eta} = 0\\ \frac{d^2 c_{n+}^*}{d\eta^2} + {}^1\!/_3\,\eta^2 \frac{dc_{n+}^*}{d\eta} - {}^{2n}\!/_3\, c_{n+}^* = c_{n-1+}^*\end{gathered} \tag{1.19}$$

where n = 2, 3, ... The initial conditions for the system of (1.15) are derived from (1.13):

$$c_{n+}^*(0) = c_{n+}^*(\infty) = 0 \tag{1.20}$$

Here n = 1, 2, 3, ... The problem of (1.19) and (1.20) has been solved by computer by a finite-difference method for n < 4; we have determined the values of the derivatives $c_{n+}^{*\prime}$ at the point $\eta = 0$ ($c_{1+}^{*\prime}(0) = 0.0990$, $c_{2+}^{*\prime}(0) = 0.0354$, $c_{3+}^{*\prime}(0) = -0.0097$), which are needed in order to calculate the pulsating mass

TABLE 1. Square of the Modulus of the Frequency Characteristic for a Planar Transducer

ω^*	$\|H^*\|^2$	ω^*	$\|H^*\|^2$	ω^*	$\|H^*\|^2$	ω^*	$\|H^*\|^2$
0	0.1112	5	0.0395	10	0.0122	15	0.0046
1	0.1035	6	0.0305	11	0.0098	16	0.0038
2	0.0852	7	0.0249	12	0.0085	17	0.0031
3	0.0674	8	0.0193	13	0.0071	18	0.0029
4	0.0520	9	0.0161	14	0.0052	19	0.0025

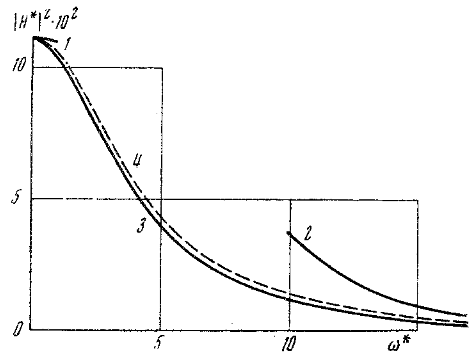

Fig. 3

flow rate to terms of order $0(\omega^{*4})$. The pulsating mass flow rate per unit surface area of the transducer is given by

$$q_- = \frac{D}{L}\int_0^L \left(\frac{\partial c_-}{\partial y}\right)_{y=0} dx = \Big(\frac{1}{3}0.806 - \frac{1}{2}\omega^{*2}c_{2+}{}^{*\prime}(0) + $$
$$+ i\,({}^3/_4\,\omega^* c_{1+}{}^{*\prime}(0) - {}^3/_8\,\omega^{*3}c_{3+}{}^{*\prime}(0))\Big)\varepsilon\exp(i\omega^* t^*)\,Dc_\infty(\tau/\mu LD)^{1/2} \quad (1.21)$$

We determine the dimensionless frequency response of the transducer as a ratio of the dimensionless pulsating mass flow rate $q_-^* = q_-/q_0$ to the dimensionless tangential stress pulsation $\tau_-^* = \tau_-/\tau_0$:

$$H^* = q_-^*/\tau_-^* = {}^1/_3 - 0.0213\,\omega^{*2} + i(-0.092\,\omega^* + 0.044\,\omega^{*3}) \quad (1.22)$$

The dimensional expression for the frequency response is found from

$$H = H^*\,q_0/\tau_0 \quad (1.23)$$

A standard theorem [6] relates the spectral densities at the input and output of a linear system to the square of the modulus of the frequency characteristic:

$$S_1(\omega)\,|H(\omega)|^2 = S_2(\omega) \quad (1.24)$$

Here S_1 is the spectral density at the input, while S_2 is that at the output; in the present case, the input signal is the friction pulsation, while the output signal is the mass flow rate pulsation. The modulus of the frequency response gives the sensitivity of the transducer as a function of frequency and is an important characteristic of an electrical diffusion transducer. For low frequencies we have

$$|H^*|^2 = ({}^1/_3 - 0.0213\,\omega^{*2})^2 + (-0.092\,\omega^* + 0.044\,\omega^{*3})^2 \quad (1.25)$$

Case of High Frequencies ($\omega^* \gg 1$). In (1.12) we can omit the term $y^*\partial c_-^*/\partial x^*$; we use for c_0^* a linear approximation within the diffusion boundary layer:

$$c_0^* = y^*\left(\frac{\partial c_0^*}{\partial y}\right)_{y^*=0} \quad (1.26)$$

which gives for c_-^* the following equation in place of (1.12):

$$i\omega^* c_-^* + \varepsilon y^{*2}\frac{\partial}{\partial x^*}\left(\frac{\partial c_0^*}{\partial y^*}\right)_{y^*=0} = \frac{\partial^2 c_-^*}{\partial y^{*2}} \quad (1.27)$$

The solution to this equation that satisfies the boundary conditions of (1.13) takes the form

$$c_-^* = \varepsilon\frac{\partial}{\partial x^*}\left(\frac{\partial c_0^*}{\partial y^*}\right)_{y=0}\left(\frac{iy^{*2}}{\omega^*} + \frac{2}{\omega^{*2}}\right) + \varepsilon\frac{\partial}{\partial x^*}\left(\frac{\partial c_0^*}{\partial y^*}\right)_{y^*=0}\frac{2}{\omega^*}\exp(-y^*\sqrt{i\omega^*}) \quad (1.28)$$

The frequency characteristic is given by the following expression up to terms of $O(\omega^{*-2})$:

$$H^* = \frac{8.68}{\omega^{*2}}(1+i) \quad (1.29)$$

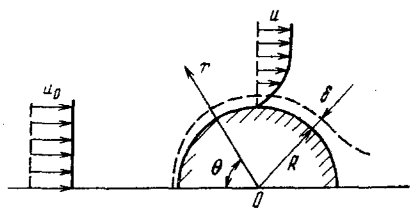

Fig. 4

The square of the modulus is

$$|H^*|^2 = 37/\omega^{*3} \tag{1.30}$$

Intermediate Frequencies [$\omega^* = O(1)$]. To determine the square of the modulus of the frequency response, we solved (1.12) subject to (1.13) by computer fitting; the numerical values are given in Table 1. A suitable approximation for $|H^{*2}|$ is given by the following formula, which coincides satisfactorily (Fig. 3) in the high- and low-frequency regions:

$$|H^*|^2 = ((9 + 0.54\,\omega^{*2})^2 + (0.27\,\omega^{*3})^2)^{-1/2} \tag{1.31}$$

From this $|H^*|^2$ and the observed S_2 one can get S_1 by employing (1.24).

We see from (1.31) that the sensitivity of such a transducer decreases exponentially as the frequency rises, but the passband can be increased by reducing the size of the electrode, as follows from the form of the dimensionless frequency.

2. Frequency Response of a Cylindrical Transducer for Velocity Pulsations

Consider the diffusion to a thin unbounded cylinder in a pulsating flow (Fig. 4); the velocity at infinity is normal to the generator of the cylinder and takes the form

$$\mathbf{u}_\infty = \mathbf{u}_0 (1 + \varepsilon \exp(i\omega t)) \tag{2.1}$$

where $\mathbf{u}_0$ is the steady-state component of the velocity; it has been shown [7] that the current function in this case is given by

$$\psi = \frac{u_0}{2}\left(2\frac{r}{R}\ln\frac{r}{R} - \frac{r}{R} + \left(\frac{r}{R}\right)^{-1}\right)\sin\theta\,(f_0 + \varepsilon f_1 \exp(i\omega t)) \tag{2.2}$$

where r and θ are polar coordinates with the axis $\theta = \pi$ in the direction of the free flow, and R is the radius of the cylinder;

$$f_0 = \left(\ln\frac{7.938}{2\mathrm{Re}}\right)^{-1}, \quad f_1 = \left(\ln\frac{\xi}{2\mathrm{Re}}\right)^{-1}, \quad \xi = 8\exp(-0.5772)\,\alpha^{-\beta},\ \beta = \frac{\alpha^2}{\alpha^2 - 1}$$

$$\alpha^2 = 1 + i\frac{\omega\nu}{u_0^2}, \quad \mathrm{Re} = \frac{Ru_0}{\nu} \ll 1, \quad \text{Re is Reynolds number}$$

We assume [5] that the velocity distribution is linear in the region of the diffusion boundary layer. We expand the current function near $r/R = 1$ and retain only the first term in the series to get

$$\psi = F_0 y^2 \sin x \tag{2.3}$$
$$(F_0 = u_0(f_0 + \varepsilon f_1 \exp(i\omega t)),$$
$$y = r/R - 1,\ x = \theta)$$

We get the longitudinal and normal components of the velocity from (2.3):

$$u = \frac{\partial\psi}{\partial y} = 2F_0 y \sin x, \quad v = -\frac{\partial\psi}{\partial x} = -F_0 y^2 \cos x \tag{2.4}$$

The equation for diffusion in the boundary-layer approximation is as follows, together with the boundary conditions in x, y coordinates:

$$\frac{\partial c}{\partial t} + \frac{F_0}{R}\left(2y\sin x\,\frac{\partial c}{\partial x} - y^2\cos\frac{\partial c}{\partial y}\right) = \frac{D}{R^2}\frac{\partial^2 c}{\partial y^2} \tag{2.5}$$

$$c(x, \infty, t) = c_\infty, \quad c(\pi, y, t) = c(x, 0, t) = 0 \tag{2.6}$$

TABLE 2. Square of the Modulus of the Frequency Characteristic for a Cylindrical Transducer

ω^*	$\|H^*\|^2$	ω^*	$\|H^*\|^2$	ω^*	$\|H^*\|^2$	ω^*	$\|H^*\|^2$
0	0.235	1.2	0.146	2.4	0.057	3.6	0.023
0.2	0.231	1.4	0.126	2.6	0.049	3.8	0.020
0.4	0.221	1.6	0.109	2.8	0.042	4	0.017
0.6	0.206	1.8	0.093	3	0.036	5	0.009
0.8	0.187	2	0.079	3.2	0.031	6	0.006
1	0.167	2.2	0.067	3.4	0.026	8	0.002

We seek the solution to (2.5)-(2.6) in the form

$$c = c_0 + \varepsilon \frac{f_1}{f_0} \exp(i\omega^* t)\, c_- \left(\omega^* = \omega \frac{\nu \Pr^{1/3} \mathrm{Re}^{4/3}}{u_0^2 f_0^{2/3}}, \ \Pr = \nu / D\right) \tag{2.7}$$

Pr is the Prandtl diffusion number. We substitute (2.7) into (2.5) and introduce the new variable

$$\eta = y \sqrt{2 \sin x} \left(3 \Pr \mathrm{Re} f_0 \int_0^x \sqrt{2 \sin y}\, dy\right)^{-1/3}$$

to get for c_0 the equations

$$\frac{\partial^2 c_0}{\partial \eta^2} + \eta^2 \frac{\partial c_0}{\partial \eta} = 0, \quad c_0(0) = 0, \quad c_0(\infty) = c_\infty \tag{2.8}$$

The solution is given by

$$c_0 = c_\infty \int_0^\eta \exp(-x^3/3)\, dx \Big/ \int_0^\infty \exp(-x^2/3)\, dx \tag{2.9}$$

The steady-state mass flow rate per unit surface area at the transducer is

$$q_0 = \frac{D c_\infty}{\pi} \int_0^\pi \left(\frac{\partial c_0}{\partial y}\right)_{y=0} dx = 1.158\, D c_\infty (\Pr \mathrm{Re} f_0)^{1/3} \tag{2.10}$$

For c_- we get by substitution into (2.5) of the new variable $y_1 = y(\Pr \mathrm{Re} f_0)^{1/3}$ and from (2.7) that

$$i\omega^* c_- + 2y_1 \sin x \frac{\partial c_-}{\partial x} - y_1^2 \cos x \frac{\partial c_-}{\partial y_1} + \frac{\partial^2 c_0}{\partial y_1^2} = \frac{\partial^2 c_-}{\partial y_1^2} \tag{2.11}$$

The boundary conditions for this equation are

$$c_-(x, \infty) = c_-(\pi, y_1) = c_-(x, 0) = 0 \tag{2.12}$$

We find the solution to (2.11) and (2.12) for the case of small, large, and moderate pulsation frequencies.

Case of Small Pulsation Frequencies ($\omega^* \ll 1$). We seek the solution in the form of the series

$$c_- = \sum_{n=0}^{\infty} (i\omega^*)^n c_{n-} \tag{2.13}$$

For c_{0-} we get from (2.11) an equation whose solution takes the simple form

$$c_{0-} = {}^1/_3\, y_1\, \partial c_0 / \partial y_1 \tag{2.14}$$

We substitute into (2.11) a new variable

$$y_2 = y_1 \sqrt{2 \sin x}, \quad x_2 = \int_0^x \sqrt{2 \sin y}\, dy$$

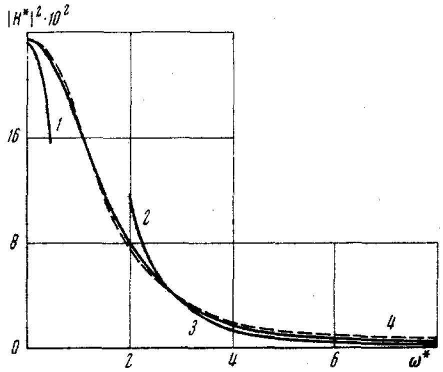

Fig. 5

and the series of (2.13) to get a system of recurrence equations

$$2\sin x_2\left(y_2\frac{\partial}{\partial x_2}-\frac{\partial^2}{\partial y_2^2}\right)c_{n-}=c_{n-1-} \tag{2.15}$$

where n = 1, 2, 3, ... The Green's function for the differential operator

$$p=y_2\frac{\partial}{\partial x_2}-\frac{\partial^2}{\partial y_2^2}$$

has been defined [8] and takes the form

$$G(x_2,y_2;x,y)=\frac{(y_2y)^{1/2}}{3}\exp\left(-\frac{y_2+y}{9(x_2-x)}\right)I_{1/3}\left(\frac{2(y_2y)^{3/2}}{9(x_2-x)}\right) \tag{2.16}$$

Here $I_{1/3}$ is a Bessel function. We solve (2.15) via (2.16) to get

$$c_{n-}=2\int_0^{x_2}\int_0^{\infty}G\sin x\, c_{n-1-}\,dx\,dy \tag{2.17}$$

We define the dimensionless frequency response as the ratio of the dimensionless mass flow rate pulsation $q_-^* = f_1 q_-/q_0 f_0$ to the dimensionless velocity pulsation $u_-^* = u_-/u_0$

$$H^*=1.82\,q_-^*/u_-^* \tag{2.18}$$

The square of the modulus of the frequency response is given to terms of $O(\omega^{*3})$ by

$$|H|^2=\left[\int_0^{x_0}\left(\frac{\partial c_{0-}}{\partial y_2}\right)_{y_2=0}dx_2\right]^2+\omega^{*2}\left\{2\int_0^{x_0}\left(\frac{\partial c_{0-}}{\partial y_2}\right)_{y_2=0}dx\int_0^{x_0}\left(\frac{\partial c_{2-}}{\partial y_2}\right)_{y_2=0}dx_2-\right.$$
$$\left.-\left[\int_0^{x_0}\left(\frac{\partial c_{1-}}{\partial y_2}\right)_{y_2=0}dx_2\right]^2\right\}=0.231-1.81\,\omega^{*2}+O(\omega^{*3}) \tag{2.19}$$
$$\left(x_0=\int_0^{\pi}\sqrt{2\sin x}\,dx=3.38\right)$$

The Mir-1 computer was used to calculate the integrals in (2.19).

Case of High Frequencies ($\omega^*\gg 1$). In (2.11) we retain the second derivatives and a term containing the large parameter ω^*; for c_- we get

$$i\omega^*c_-+\frac{\partial^2 c_0}{\partial y^2}=\frac{\partial^2 c_-}{\partial y^2} \tag{2.20}$$

whose solution that satisfies (2.12) takes the form

$$c_-=\frac{1}{2\sqrt{i\omega^*}}\left\{\int_0^{\infty}\frac{\partial^2 c_0}{\partial y^2}\exp(-\sqrt{i\omega^*}\,y)\,dy-\exp(-\sqrt{i\omega^*}\,y)\int_0^{y}\frac{\partial^2 c_0}{\partial y^2}\exp(\sqrt{i\omega^*}\,y)\,dy\right.$$
$$\left.+\exp(\sqrt{i\omega^*}\,y)\int_{\infty}^{y}\frac{\partial^2 c_0}{\partial y^2}\exp(-\sqrt{i\omega^*}\,y)\,dy\right\} \tag{2.21}$$

The square of the modulus of the frequency characteristic is

$$|H^*|^2=0.905/\omega^{*3}+O(\omega^{*-4.5}) \tag{2.22}$$

Case of Moderate Frequencies ($\omega^* = O(1)$). In this case the problem of (2.11) and (2.12) was solved by means of BÉSM-6 computer; Table 2 gives the results, which fit well with the values obtained for low and high-frequency approximations (Fig. 5). The values of $|H^*|^2$ may be approximated by

$$|H^*|^2 = 0.235 / (1 + 0.312\,\omega^{*2} + 0.103\,\omega^{*3}) \tag{2.23}$$

The dimensional expression for the square of the modulus of the transfer function is obtained from

$$|H|^2 = |H^*|^2\, q_0^2/u_0^2 \tag{2.24}$$

The spectral density known from experiment for the mass fluctuations $S_2(\omega)$ may be used with the square of the modulus for the frequency response to determine $S_1(\omega)$ from

$$S_1(\omega) = S_2(\omega) / |H(\omega)|^2 \tag{2.25}$$

The result of (2.24) shows that the sensitivity of the cylinder method falls as the frequency rises at the same rate as does that of a planar transducer; the form of the dimensionless frequency shows that the frequency passband can be increased by reducing the diameter of the wire.

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